Induction

Summation review:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \dots a_n$$

↓ formula that depends on i

$$\sum_{i=1}^{n} 1 = n \cdot 1 = n$$

Certain sums have known closed forms

↳ succint formula

KNOW THESE

$$\sum_{k=0}^{n} r^k = \frac{r^{n+1} - 1}{r - 1}$$

$$\sum_{k=1}^{5} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \dots \frac{1}{5}$$

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$$

how would we prove this for all values of $n \geq 1$?

induction is a proof technique to prove claims ($P(n)$) for some subset of integers n

Induction Outline

could change

Claim: $P(n)$ is true for all positive integers n

Proof by induction on n.

1. Base case: show $P(1)$ is true.
2. Assume strong IH: suppose $P(n)$ is true for $n = 1 \ldots k-1$ $\quad k \geq 2$

inductive hypothesis

bounds

3. Prove $P(k)$ is true, with the assumption of IH.

$P(1)$ ✓

$P(1) \wedge P(2) \wedge \ldots P(k-1) \rightarrow P(k)$ induction gives us this implication as fact

Say we let $k = 2$,

$P(1) \rightarrow P(2)$

let $k = 3$

$P(1) \wedge P(2) \rightarrow P(3)$

$\vdots$

induction tells us $P(n)$ is true for all $n \geq 1$

example: Claim: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ for all $n \geq 1$.

Proof by induction on n.

① Base case: is true that $\sum_{i=1}^{1} i = \frac{1(1+1)}{2}$

$1 = 1$ ✓ base case holds

② IH: Suppose $\underbrace{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}_{\text{claim}}$ for $\underbrace{n = 1 \ldots k-1}_{\text{bounds}}$

③ Inductive step:

Goal: Show that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ for $n = k$

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

work w/ LHS until get RHS

$$\sum_{i=1}^{k} i = \left(\sum_{i=1}^{k-1} i\right) + k$$

Note

$$\sum_{i=1}^{k} i = \underbrace{1+2+\ldots(k-1)}_{\sum_{i=1}^{k-1} i} + (k)$$

By IH, $\sum_{i=1}^{k-1} i = \frac{(k-1)k}{2}$

So, $\sum_{i=1}^{k} i = \frac{(k-1)k}{2} + k = \frac{k^2-k}{2} + \frac{2k}{2} = \frac{k^2+k}{2} = \frac{k(k+1)}{2}$

That is what we wanted to show, so the claim is true for $n=k$, and therefore all positive integers